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A NEW METHOD FOR DELAY-MODULATING A PULSE TRAIN

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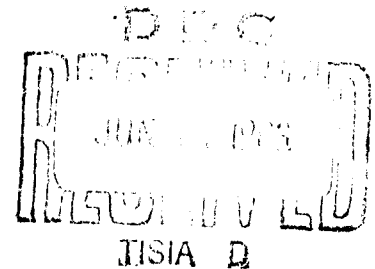
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Abstract

A new circuit for obtaining pulse frequency modulation (which is equivalent to pulse delay modulation) is derived. It is shown that the frequency is continuously variable over a considerable range. A practical implementation of the circuit is shown. Methods of improving the linearity of the device are presented. In theory, these techniques can be made to yield a circuit having no non-linearity.

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The author has recently been faced with the problem of finding (or designing) circuitry whose output is a train of pulses of variable delay. At first the solution seems simple: Use the output of an FM signal generator to key a pulse generator (e. g. , a monostable multivibrator) and vary the pulse delay by phase-modulating the output of the signal generator. Thus if the waveform of the signal generator is

$$\sin(\omega_c t - d\theta(t)) \quad ,$$

and if the pulse generator triggers when the phase of signal output of the generator reaches ϕ , the time of occurrence of n^{th} pulse, t_n , is given by the relation

$$\omega_c t_n - d\theta(t_n) = 2\pi n + \phi \quad . \quad (1)$$

The delay imparted to this pulse by the modulation is therefore

$$\frac{d\theta(t_n)}{\omega_c} \quad .$$

If $\theta(t) = \sin \omega_m t$, then the parameter d becomes identical with the familiar deviation ratio. Taking ω_c equal to $2\pi \times 500,000$ and the maximum desired delay at $200 \mu \text{ sec}$, we find that

$$d_{\text{max}} = 628 \quad .$$

A brief survey of the literature has shown that deviation ratio of this order of magnitude is unattainable using simple, present-day circuitry. Modulators of the Armstrong type are only capable of providing deviation ratios of the order of unity while the solid state counterpart of the reactance tube, the voltage variable capacitor, is so drastically non-linear that its use seems limited to low deviation ratio circuits. In order to achieve a high deviation ratio with one of the techniques listed above, it is apparent that one must first modulate at a low deviation ratio and pass the resultant waveform through a sufficient number of frequency multipliers. If a low center frequency is desired, the output of the last multiplier unit must be mixed with a sinusoid, the frequency of which is stable. This is the essential principle of a commercially available signal generator. It is clearly an unattractive solution for the problem at hand.

It is apparent that an alternate scheme must be considered. We treat the problem here in the time domain and so base our analysis on wave-shaping techniques; thus it is our intent to generate pulses directly in accordance with Eq. (1). To begin, Eq. (1) is rewritten as

$$\omega_c [t_{n+1} - t_n] - d[\theta(t_{n+1}) - \theta(t_n)] = 2\pi \quad . \quad (2)$$

However, since $\theta(t)$ is an audio bandwidth process, we may approximate the difference by the derivative multiplied by the difference of the arguments.*

*We assume the $\theta(t)$ is differentiable. If this is so, by the law of the mean

$$\theta(t_{n+1}) - \theta(t_n) = \dot{\theta}(t') [t_{n+1} - t_n]$$

where $t_n < t' < t_{n+1}$. Since $\theta(t)$ is an audio bandwidth process, $\dot{\theta}(t')$ does not differ appreciably from $\dot{\theta}(t_{n+1})$.

Consequently, Eq. (2) becomes

$$\omega_c [t_{n+1} - t_n] - d\dot{\theta}(t_{n+1})[t_{n+1} - t_n] = 2\pi \quad (3)$$

or

$$t_{n+1} - t_n = \frac{2\pi}{\omega_c - d\dot{\theta}(t_{n+1})} \quad (4)$$

Now $d\dot{\theta}(t_n)$ is the instantaneous frequency deviation from ω_c . Since this is usually small compared with ω_c , we have the approximation

$$t_{n+1} - t_n = \frac{2\pi}{\omega_c} \left[1 + \frac{d}{\omega_c} \dot{\theta}(t_{n+1}) \right] \quad (5)$$

Equation (5) is realizable by means of the circuit shown in block diagram form in Fig. 1. The circuit operates as follows: Suppose a pulse occurs at time t_n (see Fig. 2). This pulse resets the sweep circuit and so initiates a sweep of slope m , q seconds later. When the height of the sweep equals $a\dot{\theta}(t) + b$, the comparator excites the pulse former, and the next pulse appears. Hence

$$t_{n+1} - t_n = q + \frac{a\dot{\theta}(t_{n+1}) + b}{m} = q + \frac{b}{m} + \frac{a}{m} \dot{\theta}(t_{n+1}) \quad (6)$$

Thus if

$$q + \frac{b}{m} = \frac{2\pi}{\omega_c} \quad (7)$$

and

$$\frac{a}{m} = \frac{2\pi d}{\omega_c^2} \quad (8)$$

it follows that the time interval between the pulses is as stated by Eq. (5).

Note that the comparator must be inactive during interval q .

A practical realization of the above scheme appears in Fig. 3. negative-going sweep is applied to the emitter of the blocking oscillator transistor. When modulating voltage at the output of emitter follower exceeds this voltage, the blocking oscillator fires. The transistor in the reset circuit goes into clamp on the backswing of the pulse of the blocking oscillator, thereby resetting the circuit. The blocking oscillator cannot fire while the sweep is being reset, since this operation cuts off the transistor in this circuit. The variation of pulse repetition rate with modulating voltage is plotted in Fig. 4. It is seen that the pulse frequency is continuously variable from 200 kcps to 1370 kcps.

It is of interest to compare the pulse delay circuit with the solid state equivalent of the reactance tube modulator with respect to their capabilities in producing linear FM. Adopting a viewpoint from FM theory, the instantaneous frequency of the output of the pulse delay circuit, f_{pd} , is

$$f_{pd} = \frac{1}{t_{n+1} - t_n} = \frac{1}{\frac{2\pi}{\omega_c} + \frac{2\pi d}{\omega_c^2} \theta(t)} \quad (9)$$

by Eqs. (6), (7), and (8). Consequently, to a second-order approximation

$$f_{pd} = f_c \left\{ 1 - \frac{d}{\omega_c} \theta(t) + \left[\frac{d}{\omega_c} \theta(t) \right]^2 \right\} \quad (10)$$

where f_c is the center frequency. If we define the figure of merit, ρ , as the ratio of the second order term to the first, then

$$\rho_{pd} = \frac{\Delta f}{f_c} \quad (11)$$

where Δf is the frequency deviation given by the first order term. (Note that ρ is essentially the ratio of the error in linearity to the frequency deviation.) Now the capacity of a voltage variable capacitance diode varies according to the relation¹

$$C(V) = \frac{k}{(\phi - V)^n} \quad (12)$$

where $0.3 < n < 0.5$. (The quantity n is a function of the construction of the diode.) It can be shown that if the frequency deviation is very small compared with the center frequency, then the figure of merit for a reactance modulator, ρ_{rm} , is

$$\rho_{rm} = 3/2 \frac{\Delta f}{f_c} \quad (13)$$

This condition, however, is not the one anticipated: The frequency deviation will be an appreciable fraction of the center frequency and so ρ_{rm} will be much worse. It follows that the pulse delay circuit is superior to the reactance modulator under all conditions.

Suggestions for Future Work

It is apparent that linearity of the pulse delay circuit can be improved by insisting that the time between pulses obeys Eq. (4). Thus the problem arises of synthesizing pulse circuits in which the time between pulses obeys an equation of the form

$$t_{n+1} - t_n = G[\theta(t_{n+1})] \quad (14)$$

The block diagram of such a circuit is shown in Fig. 5. It is easily seen that we may make $t_{n+1} - t_n$ equal to any functional of θ , evaluated at t_{n+1} . Thus we are led to consider a family of modulation schemes which are generalizations of delay modulation or phase modulation. These schemes are all feasible. It is also clear that by proper filtering, high deviation ratio FM may be obtained from this circuit. The experimental results indicate that this is correct. When the output of the circuit is mixed (in the limiter stage of an FM receiver) with a 10.2 Mcps note and detected in the discriminator of the receiver, the modulation is clearly audible. (The filtering here is provided by the receiver's bandpass characteristic.) Indeed, the higher harmonics of the output (around 88-100 Mcps) have been detected on an FM receiver.

It should be noted that the method of linearity improvement given above has no theoretical bound (i. e., can be made to yield perfect linearity) since it depends on the mathematical foundation of the modulation scheme. This is not true for other schemes of obtaining FM where the frequency depends on a parameter of a device which changes from device to device (e. g., the g_m of a tube).

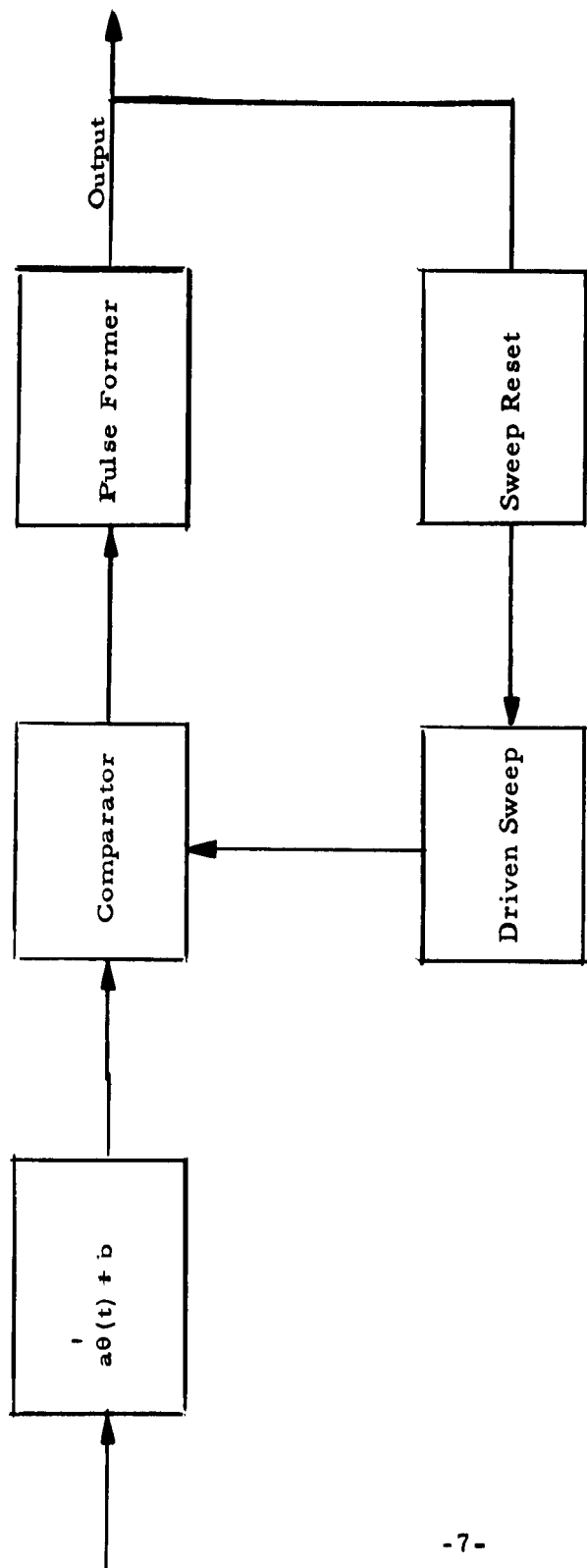


Fig. 1

Pulse Delay Circuit

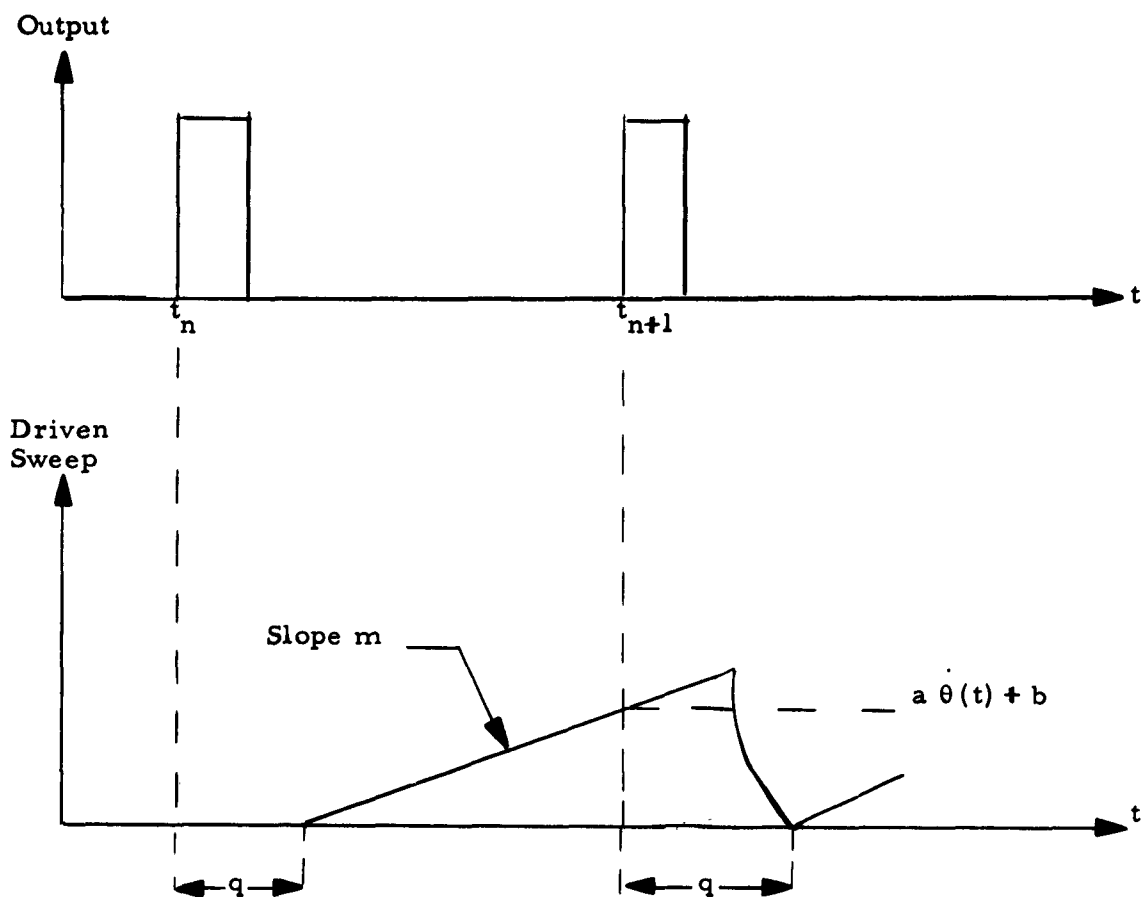


Fig. 2

Waveforms Illustrating the
Operation of the Pulse Delay Circuit

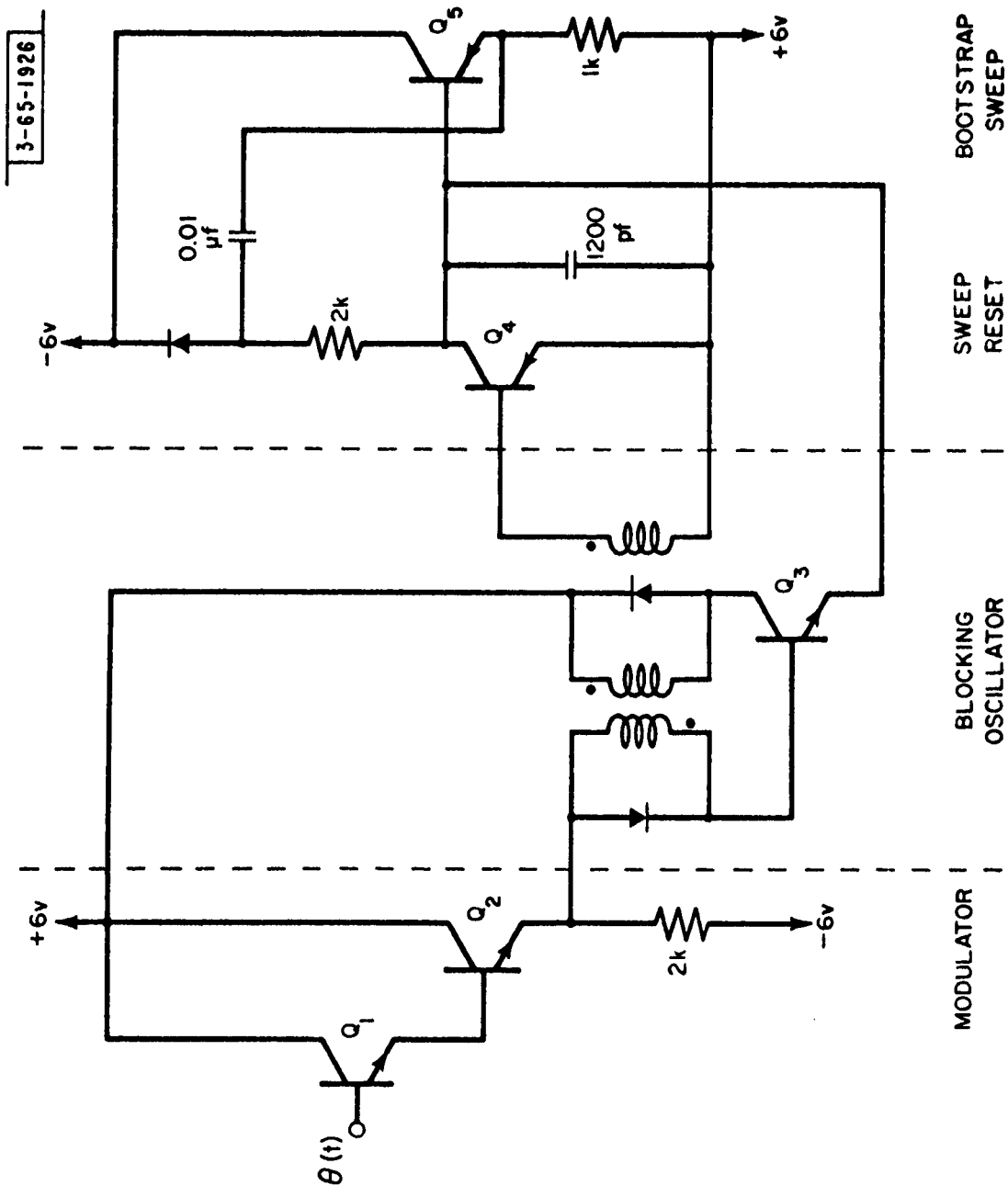


Fig. 3
A Practical Pulse Delay Circuit

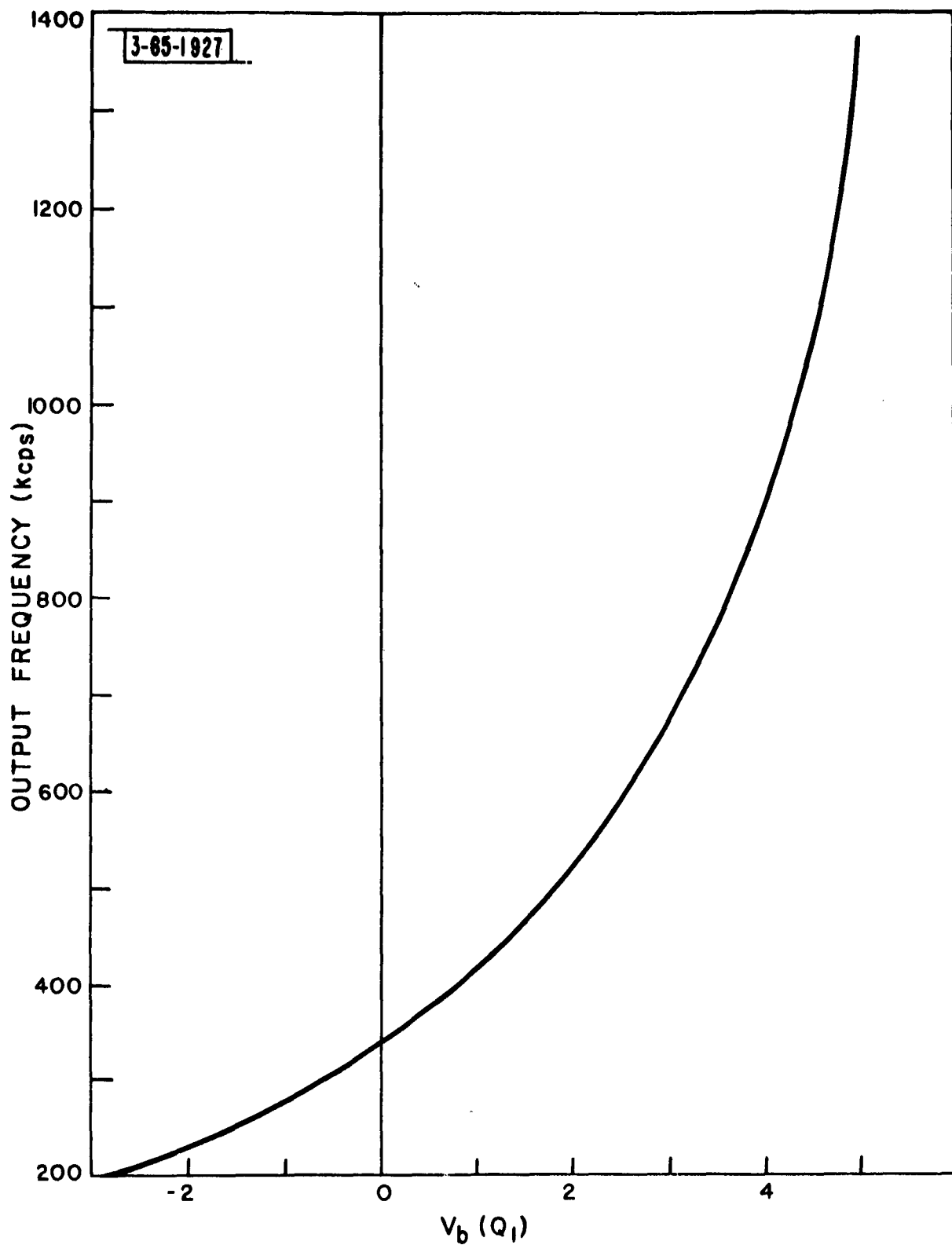


Fig. 4

Pulse Repetition Rate vs Modulating
Voltage for Circuit Shown in Fig. 3.

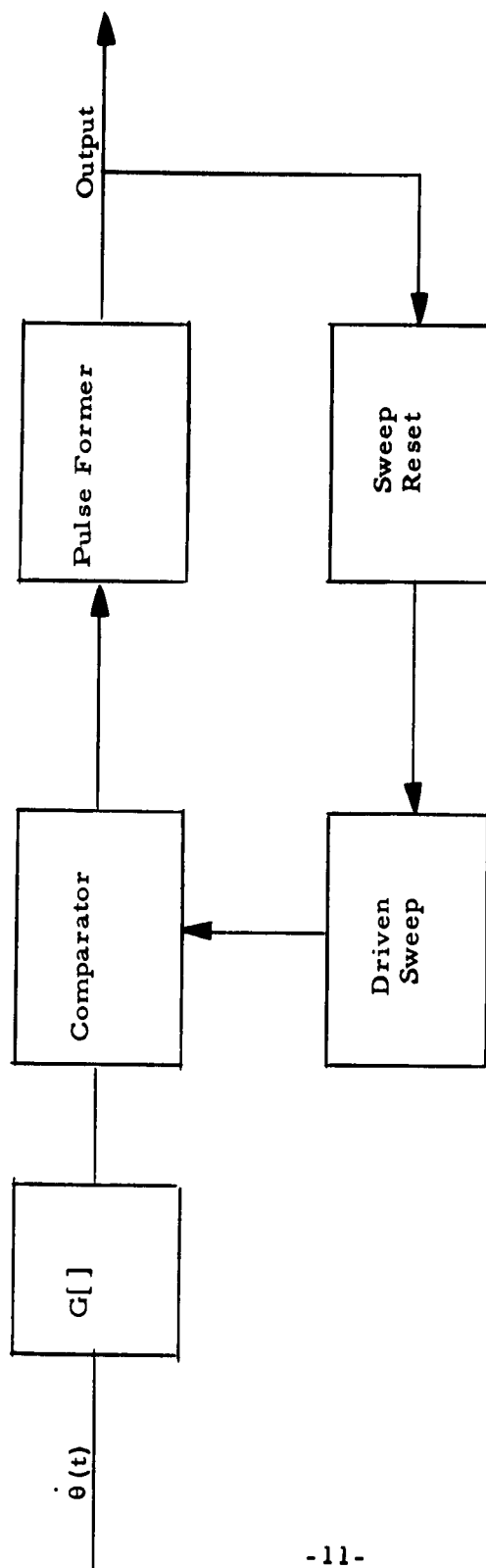


Fig. 5

A Circuit for Realizing the Equation

$$t_{n+1} - t_n = G[\dot{\theta}(t_{n+1})]$$

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The author wishes to thank Charles Hoover for his capable work in breadboarding and testing the circuit shown.

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